Reduction in Belief Elicitation

Andrew Dustan, Kristine Koutout, & Greg Leo

August 2022

Abstract

The state-of-the-art in eliciting probabilistic beliefs, the Binarized Quadratic Scoring Rule (BQSR), relies on an easily overlooked preference assumption: the reduction of compound lotteries. In a lab experiment, we find evidence that 70% of people violate the reduction assumption for at least some compound lotteries involved in the BQSR. We show that people whose preferences are consistent with reduction are 33% more likely to report accurate beliefs compared to those whose preferences are not consistent with reduction. We implement a novel Rank-Ordered Elicitation (ROE), which does not rely on the reduction of compound lotteries, to test whether a procedure that eliminates the need for reduction increases the accuracy of reported beliefs. We find no evidence for this last hypothesis. Further exploration shows that, as theoretically predicted, the difference between reducers and non-reducers is sharply attenuated in the elicitation of a 50% probability. Taken together, the results suggest that preferences inconsistent with the reduction of compound lotteries are an important driver of the empirical under-performance of the BQSR, while leaving open a role for other factors in reducing this method's behavioral incentive compatibility.

1 Introduction

Beliefs play an important role in decision-making under uncertainty; however, beliefs are not directly observable. In economics, we generally infer beliefs from choices, but choices are determined both by beliefs and preferences. Thus, any procedure for uncovering beliefs must make important assumptions about the structure of preferences.

The current state-of-the-art in eliciting probabilistic beliefs is the binarized quadratic scoring rule (BQSR) (Hossain and Okui, 2013). The common understanding of the BQSR is that it is incentive-compatible for all expected utility maximizers, as well as some non-expected utility maximizers. The BQSR pays people in compound lotteries, relying on an easily overlooked assumption about preferences: the reduction of compound lotteries.

In this paper, we test empirically whether 1) this reduction assumption is violated by participants for the types of compound lotteries involved in the BQSR, 2) the failure of reduction predicts accuracy in the BQSR, and 3) a procedure that does not require reduction elicits more accurate beliefs from non-reducers. We conduct these tests using a simple, two-stage experiment. In the first stage, participants make choices between two compound lotteries. In the second stage, participants report their beliefs about objective probabilities. One of the compound lotteries in each of the first-stage choices corresponds to the compound lottery the participant faces *if they choose to accurately report* the objective probability elicited using the BQSR. In addition, we elicit corresponding objective probabilities using a novel procedure, the Rank-Ordered Elicitation (ROE), which does not require the reduction of compound lotteries.

We find that most participants do not consistently reduce compound lotteries and that this behavior predicts inaccuracy in the BQSR; however, non-reduction also predicts inaccuracy in the ROE, which does not rely on reduction. Only 30% of participants are always "reducers," meaning that they consistently choose the compound lottery with the highest expected payoff. In line with theory, participants whose preferences are consistent with reduction in the relevant lottery are 13 percentage points more likely to report the corresponding objective probability accurately than those who choose the compound lottery with the lower expected payoff. Unexpectedly, this difference in accuracy persists in the ROE. Indeed, the BQSR and ROE perform similarly to each other for both reducers and non-reducers. We further test for the role of reduction in the BQSR by comparing reducers to non-reducers when reporting a 50% probability. As theoretically predicted, there is no detectable difference between the performance of reducers and non-reducers in reporting the centered probability.

In summary, reduction plays an important role in the incentive compatibility of the BQSR, but removing the need for that assumption in another procedure does not resolve the issue. These results highlight the need to expand our selection criteria. Behavioral economics establishes the roles of attention, heuristics, and many other biases in economic decision-making. In line with Danz et al. (2022), we suggest that belief elicitation procedures should account for these behavioral factors, in addition to theoretical incentive compatibility.[1]

We consider elicitations of the form "what is the probability that event E occurs?" For any reported belief $\tilde{p}$ about this probability, the BQSR results in a compound lottery. If the event occurs, the participant is paid a monetary reward with the probability $1 - (1 - \tilde{p})^2$. If the event does not occur, the participant receives the monetary reward with probability $1 - \tilde{p}^2$.

Without an assumption of reduction, there is no guarantee that the compound lottery the participant is compensated with under truth-telling is most-preferred among all other available compound lotteries. For instance, the compound lottery that results from reporting a belief of 50% pays the monetary reward with 75% probability

[1] Danz et al. (2022) refer to this holistic criterion as "behavioral incentive compatibility".

regardless of whether the event occurs. Consider choosing between that lottery that pays a 75% chance with 100% chance, and the compound lottery for accurately reporting a belief of $\frac{1}{3}$: 33% chance of a 56% chance and 67% chance of an 89% chance. The latter lottery pays a higher expected simple lottery (78%), but it is reasonable to believe a participant would in fact prefer the lottery that does not correspond to accurate reporting.

To avoid the reduction assumption on compound lotteries, we develop a new procedure to elicit beliefs: the ROE. The ROE draws inspiration from multiple price lists by asking participants to rank an option in which payment is based on whether the event E of interest occurs relative to a list of objective lotteries that pay with known probabilities. The intuition for this procedure is that, if a participant prefers to be paid based on whether E occurs to being paid with 50% probability, then the participant believes there is a greater than 50% probability that E occurs.

We make three contributions to the literatures on reduction and belief elicitation. First, we document that a majority of people have preferences inconsistent with reduction of the *specific* compound lotteries used to incentivize accurate reporting in the BQSR. Concurrently, Danz et al. (2022) also find that people often do not choose the compound lottery that maximizes expected payoff in lotteries relevant to the BQSR. We make an additional contribution through our intra-participant design by providing the first evidence that people who fail to reduce are less likely to report accurate beliefs than those who do not. This difference is mitigated in the elicitation of a centered probability (50%), affirming the importance of the reduction assumption in the BQSR. We then show that addressing the failure of the reduction assumption theoretically using the ROE does not increase accurate reporting.

A sizable empirical literature explores the plausibility of the reduction of compound lotteries axiom, dating at least to Bar-Hillel (1973). As Hajimoladarvish (2018) summarizes, reduction has been tested either by comparing choices between compound lotteries and their equivalent simple lotteries (Harrison et al., 2015), or by eliciting and comparing certainty equivalents for both compound and simple lotteries (Abdellaoui et al., 2015). Support for the axiom is mixed, with findings seemingly sensitive to framing (Bernasconi and Bernhofer, 2020), details of the experimental design (Harrison et al., 2015), and the method of comparison (Hajimoladarvish, 2018). Our finding that many participants behave inconsistently with the reduction of compound lotteries is thus not at odds with existing literature, though it is the first to evaluate reduction as it relates to accurate reporting in the BQSR.

The use of probability currency to induce incentive compatibility in belief elicitation for people with non-risk-neutral preferences dates to the 1960s (Smith, 1961; Roth and Malouf, 1979). Hossain and Okui (2013) were the first to generalize the method of binarizing *any* proper scoring rule to achieve incentive compatibility for a wide range of preferences. Since then, the BQSR has been used in many applications (see, for example, Babcock et al., 2017 or Dianat et al., 2018). Recently, Danz et al. (2022) and Healy & Kagel (2021) identify issues with eliciting beliefs, raising questions about the empirical performance of the BQSR. We test one explanation for less than theoretically-optimal performance of the BQSR: preferences inconsistent with the reduction of compound lotteries.

The remainder of the paper is structured as follows. Section 2 gives a theoretical treatment of the BQSR and ROE, showing that the BQSR requires reduction for incentive compatibility while the ROE does not. Section 3 presents the experimental design. Section 4 reports the main findings and the results of pre-specified hypotheses. Section 5 concludes.

2 Theory

In this section, we first provide the precise preference assumptions under which the Binarized Quadratic Scoring Rule (BQSR) is incentive-compatible. Then, we present the Rank-Ordered Elicitation (ROE) for eliciting beliefs, which builds on the use of multiple price lists. Lastly, we show that the ROE requires a weaker assumption than reduction to be incentive-compatible.

2.1 Binarized Quadratic Scoring Rule

The BQSR is part of a longer tradition in experimental economics of using probabilistic currency to linearize preferences. As shown in Hossain and Okui (2013), binarizing any proper scoring rule results in a procedure that yields incentive-compatible elicitation of the target statistic under any expected utility preferences (regardless of risk preferences) and some non-expected utility preferences.

The stated key assumption in Hossain and Okui is monotonicity over binary objective lotteries. That is, a participant prefers a simple lottery over outcomes a and b to another simple lottery over those outcomes if and only if it yields the better outcome more often. Formally, let $L(p)$ be the binary lottery that yields outcome a with probability p and outcome b with probability $1-p$ and assume $a \succ b$. The key assumption is that preferences over simple lotteries can be represented by a preference functional $V(\cdot)$ with the property that $V(L(p)) > V(L(p^*)) \Leftrightarrow p > p^*$.

Theorem 1 of Hossain and Okui states that under suitable requirements on the underlying scoring rule, the optimal action taken by a participant meeting this key assumption in the binarized version of the scoring rule will be the same as the action taken by a risk-neutral participant when incentivized by the standard non-binarized version of the rule. Thus, any scoring rule that is incentive compatible for a risk-neutral participant can be extended through binarization to be incentive compatible regardless of risk prefernces.

While the monotonicity assumption on binary objective lotteries is key for the incentive compatibility of the BSR, it is not sufficient. As Hossain and Okui note, they "also assume that the preference of the agent under the BSR depends only on the probability of winning." Implicit in this assumption is that a participant's preferences over the *compound lotteries* that yield outcome a with *expected* probability p are the same as their preferences over *simple lotteries* that yield outcome a with probability p. To demonstrate the need for reduction in BSR more concretely, we formalize our analysis by first presenting some notation.

2.1.1 Environment

We assume that participants make choices on the basis of well-defined subjective probabilities about events, but do not necessarily have preferences consistent with expected-utility. The environment follows some of the conventions in Machina (1995). Our analysis of the BQSR makes use of the following objects and types of lotteries:

The set of *outcomes* is $\mathscr{X} = \{a, b\}$.[2] The set of *events* are $E \in 2^{\mathscr{S}}$, in which $\mathscr{S} = \{..., s, ...\}$ are states of the world. *Simple Lotteries* are objective lotteries of the form $S = (a \circ p, b \circ (1-p))$, where $p \in [0,1]$. *Compound Lotteries* are objective lotteries of the form $C = (S_1 \circ q, S_2 \circ (1-q))$, where $q \in [0,1]$ and S_1, S_2 are simple lotteries. *Compound Subjective Lotteries* mix objective probabilities and subjective events and are of the form $H = (S_1 \circ E, S_2 \circ E^c)$ where S_1, S_2 are simple lotteries, E is an event and E^c is the complement of that event. Finally, the set of *the set of all Compound Subjective Lotteries* is $\mathscr{H}$.

2.1.2 Incentive Compatibility and the BQSR

The BQSR asks participants to choose among a family of compound subjective lotteries indexed by the probability $\tilde{p}$ (the "guessed" probability). These lotteries have the following form:

$$H(\tilde{p}) = (S(\tilde{p}) \circ E, S(1-\tilde{p}) \circ E^c)$$

Where the simple lottery $S(\tilde{p})$ is:

$$S(\tilde{p}) = \left(a \circ 1 - (1-\tilde{p})^2, b \circ (1-\tilde{p})^2\right)$$

Incentive compatibility requires that $\forall \tilde{p} \in [0,1]/p \; H(p) \succ H(\tilde{p})$. As in Hossain and Okui (2013), the only dominance assumption we will put on lotteries is on simple lotteries. However, we will only be able to compare two lotteries through this dominance assumption if those lotteries can be mapped into simple lotteries.

Thus, a proof of incentive compatibility will require some way of mapping subjective compound lotteries into simple (objective) lotteries. This requires two things. First, the ability to eliminate the subjectivity. Second, the ability to eliminate the compounding.

With this overview, we can formally state the axioms needed for the incentive compatibility of the BQSR. Let $\succsim$ be a complete, reflexive, and transitive preference

[2] Generally, the set of outcomes is assumed to be *some* finite set, but since both of the procedures discussed in this paper use only two outcomes we have included this restriction.

relation over $\mathscr{H}$. The assumptions we make of $\succsim$ are:

Axiom 1: *Monotone on Simple Lotteries:*
For $a \succ b$, $p > p' \Leftrightarrow (a \circ p, b \circ (1-p)) \succ (a \circ p', b \circ (1-p'))$.

Axiom 2: *Subjective/Objective Replacement on Simple Lotteries:*
$\exists p \in [0,1] \;\; s.t.[S_1 \circ E, S_2 \circ E^c] \sim (S_1 \circ p, S_2 \circ (1-p))$.

Axiom 3: *Reduction:*
For any $q_1,\ q_2,\ p \in [0,1]$
$(a \circ q_1, b \circ 1 - q_1) \circ p, (a \circ q_2, b \circ 1 - q_2) \circ (1-p) \sim$
$a \circ [q_1 p + q_2 (1-p)], b \circ [(1-q_1) p + (1-q_2)(1-p)]$

We can now prove incentive compatibility.

Proof: Incentive compatibility requires:

$$\forall \tilde{p} :\in [0,1]/p \quad (S(p) \circ E, S(1-p) \circ E^c) \succ (S(\tilde{p}) \circ E, S(1-\tilde{p}) \circ E^c)$$

By Axiom 2, we can replace the subjective event with and objective probability. There exists some $p \in [0,1]$ (the subjective belief about event E) such that:

$$(S(\tilde{p}) \circ E, S(1-\tilde{p}) \circ E^c) \sim (S(\tilde{p}) \circ p, S(1-\tilde{p}) \circ (1-p))$$

This indifference can be applied to both sides of the incentive compatibility condition. Since preferences are transitive, this results in a characterization of incentive compatibility involving only objective probabilities.

$$\forall \tilde{p} :\in [0,1]/p \quad (S(p) \circ p, S(1-p) \circ (1-p)) \succ (S(\tilde{p}) \circ p, S(1-\tilde{p}) \circ (1-p))$$

Applying Axiom 3, these compound lotteries can be replaced with their induced simple lotteries. Again, by transitivity, this generates an incentive compatibility characterization involving only simple lotteries to which the dominance axiom can be applied:

$$\begin{aligned} &\left(a \circ p\left(1-(1-p)^2\right) + (1-p)\left(1-p^2\right), b \circ p(1-p)^2 + (1-p)p^2\right) \\ \succ &\left(a \circ p\left(1-(1-\tilde{p})^2\right) + (1-p)\left(1-\tilde{p}^2\right), b \circ p(1-\tilde{p})^2 + (1-p)\tilde{p}^2\right) \end{aligned}$$

Applying Axiom 1, the condition is true if and only if:

$$p\left(1-(1-p)^2\right) + (1-p)\left(1-p^2\right) > p\left(1-(1-\tilde{p})^2\right) + (1-p)\left(1-\tilde{p}^2\right)$$

Since the right side is strictly quasi-concave and maximized at $\tilde{p} = p$, the procedure is incentive compatible.[3]

[3] In addition, and more informally, it requires participants to notice or believe that $p\left(1-(1-p)^2\right) + (1-p)\left(1-p^2\right) > p\left(1-(1-\tilde{p})^2\right) + (1-p)\left(1-\tilde{p}^2\right)$ is maximized at $\tilde{p} = p$.

2.1.3 Why might the incentive compatibility of BQSR fail?

It is easy to construct preferences for which incentive compatibility fails even when the participant's preferences are monotone on binary objective lotteries (Axiom 1). These constructions involve a failure either to replace subjective events with objective probabilities (Axiom 2), or to reduce compound to simple lotteries (Axiom 3).

Every belief elicitation methodology that attempts to map subjective beliefs into an objective probability will require a comparison of subjective and objective lotteries. Thus, a failure of Axiom 2 (or some related assumption) would cause problems for any probabilistic belief elicitation. Thus, we focus on the types of preference that might violate the reduction axiom (Axiom 3).

Example: A participant believes the probability of event E is $\frac{1}{3}$. The participant is indifferent between \$10 if event E occurs and \$10 with a $\frac{1}{3}$ chance. However, the participant strictly prefers:

$$(\$10 \circ 0.75, \$0 \circ 0.25) \circ \frac{2}{3}, (\$10 \circ 0.75, \$0 \circ 0.25) \circ \frac{1}{3}$$

to

$$(\$10 \circ 0.89, \$0 \circ 0.11) \circ \frac{2}{3}, (\$10 \circ 0.56, \$0 \circ 0.44) \circ \frac{1}{3}$$

These are precisely the compound lotteries that result from reporting respectively 50% and 33.3% in the BQSR for this participant once the subjective events are replaced with the objective probabilities associated with the subject's beliefs about those events. These preferences are inconsistent with reduction and would lead the participant to prefer reporting a 50% belief in the BQSR to their true belief of (approximately) 33.3%.

An extreme form of this failure in reduction would occur if a participant's preferences over compound lotteries are such that they maximize the minimum probability of the better outcome over all contingencies in a compound lottery. In fact, this would lead a participant to *always* report 50% in the BQSR.

The preferences in this example are consistent with a form of aversion to variance *within* compound lotteries.[4] On the other hand, the ROE does not rely on the kind of compound lotteries which are problematic in these examples. Specifically, reduction is not required.

2.2 Rank-Ordered Elicitation

The Rank-Ordered Elicitation (ROE) builds on a multiple price list methodology. Informally, it draws on the following observation: if a participant prefers to be paid, for instance, a with a 50% chance to being paid a if the event E happens, but prefers to be paid if E happens to being paid with a probability 40%, then their belief about the probability that E occurs must be within the range 40% to 50%.

[4]This failure could be responsible for the pull-to-center effect found in Danz et al. (2022)

2.2.1 Description of ROE

While the BQSR "pays" in *compound subjective lotteries*, the ROE pays in something we call *simple mixtures*, which are probability distributions over *simple (objective) lotteries* and *simple subjective lotteries* (a type of lottery that does not appear in the proof of incentive compatibility for the BQSR). We define each of these as follows:

Simple Subjective Lotteries are lotteries of the form $G_E = (a \circ E, b \circ E^c)$ where E is an event and E^c is the complement of that event. *Simple Mixtures* of the form $M = (L_1 \circ p_1, L_2 \circ p_2, ..., L_n \circ p_n)$ where $p_i \in [0, 1]$ with $\sum_{i=1}^{n} p_i = 1$, and each L_i is either a simple lottery or a simple subjective lottery. *The set of all Simple Mixtures* is $\mathscr{M}$.

Formally, to elicit the probability of event E, construct a set of simple objective lotteries $\{S_{p_i}\}_{i=1}^{m}$ of the form $S_{p_i} = p_i \circ a, (1 - p_i) \circ b$ with $p_i > p_{i+1}$, in which m is the total number of objective lotteries. In addition, construct a single simple subjective lottery of the form "G_E : a if event E and b otherwise" that is $(a \circ E, b \circ E^c)$.

Eliciting a participant's belief about the event E amounts to determining the position of the lottery G_E in their linear preference ordering among the objective lotteries $\{S_{p_i}\}_{i=1}^{m}$.

To elicit the position that this lottery falls in the linear order, ask the participant to rank G_E in their preferred position among the ordered simple objective lotteries. To incentivize truthful ranking, pay the participant by randomly choosing one objective lottery S_{p_i} and paying according to whichever lottery the participant ranks higher: the objective lottery S_{p_i} or subjective lottery H. In other words, if the participant ranks S_{p_i} above G_E, pay according to S_{p_i}. Otherwise, pay according to G_E. An example is given below.

This procedure produces simple mixtures of the form: $\left(L_1 \circ \frac{1}{m}, ..., L_m \circ \frac{1}{m}\right)$ where L_i is whichever lottery from each pair S_{p_i}, G_E that the participant ranks higher.

Subjective Lottery (H)	Objective Lotteries (R_{p_i})
\$10 if Event is True	0% chance of \$10
	20% chance of \$10
	40% chance of \$10
	60% chance of \$10
	80% chance of \$10
	100% chance of \$10

Table 1: ROE Example

The participant chooses where to rank the subjective lottery on the left among the objective lotteries on the right.

2.2.2 Incentive Compatibility and the ROE

First, we state the axioms needed for the incentive compatibility of our procedure. Let $\succsim$ be a complete, reflexive, and transitive preference relation over $\mathscr{M}$. The assumptions we make of $\succsim$ are:

Axiom 1: *Monotone on Binary Objective Lotteries:*
For $a \succ b$, $p > p' \Leftrightarrow (a \circ p, b \circ (1-p)) \succ (a \circ p', b \circ (1-p'))$.

Axiom 2': *Subjective/Objective Replacement on Degenerate Simple Lotteries*[5]*:*
$\exists p \in [0,1]\ s.t.[a \circ E, b \circ E^c] \sim (a \circ p, b \circ (1-p))$

Axiom 3': *Statewise Monotone on Simple Mixtures:*
$L_i^* \succ L_i \Leftrightarrow (L_1 \circ p_1, ..., L_i^* \circ p_i, ..., L_n \circ p_n) \succ (L_1 \circ p_1, ..., L_i \circ p_i, ..., L_n \circ p_n)$

We can now prove incentive compatibility of the ROE under these axioms.

Proof: By Axiom 1, $\{S_{p_i}\}_{i=1}^m = S_{p_1} \succ S_{p_2} \succ ... \succ S_{p_m}$ is a linear order. By Axiom 2, there is some p (the subjective belief about event E) such that $G_E \sim (a \circ p, b \circ (1-p))$. Denote this objective lottery by S_p. By Axiom 1, S_p is somewhere in the linear order of $\{S_{p_i}\}_{i=1}^m$. By transitivity, G_E takes the same place in this linear order. Thus, the participant's preference relation on $\{\{S_{p_i}\}_{i=1}^m, G_E\}$ is a linear order. Let i^* be the smallest p_i such that $S_{p_i^*} \succ G_E$ and let $i = 0$ if $G_E \succ S_{p_1}$. We call i^* the participant's belief type, where a player of belief type i^* has a belief that $p \in [p_{i^*}, p_{i^*+1}]$. There are $n+1$ possible types $i^* \in (0, ..., n)$.

[5] Axiom 2' is a weakening of 2.

Let L_i be the simple mixture that results from ranking G_E between S_{p_i} and $S_{p_{i+1}}$. This is the mixture:

$$L_i = \left(S_{p_1} \circ \frac{1}{m}, ..., S_{p_i} \circ \frac{1}{m}, G_E \circ \frac{1}{m}, ..., G_E \circ \frac{1}{m} \right)$$

Incentive compatibility of the ROE requires that for a player of type i^*, $\forall i \in \{0, ..., n\}$: $L_{i^*} \succsim L_i$. Consider L_{i^*} and L_{i^*+1}. These lotteries are:

$$L_{i^*} = \left(S_{p_1} \circ \frac{1}{m}, ..., S_{p_{i^*}} \circ \frac{1}{m}, G_E \circ \frac{1}{m}, ..., G_E \circ \frac{1}{m} \right)$$

$$L_{i^*+1} = \left(S_{p_1} \circ \frac{1}{m}, ..., S_{p_{i^*}} \circ \frac{1}{m}, S_{p_{i^*+1}} \circ \frac{1}{m}, G_E \circ \frac{1}{m}, ..., G_E \circ \frac{1}{m} \right)$$

These differ by a single replacement of G_E with $S_{p_{i^*+1}}$. Since $G_E \succsim S_{p_{i^*+1}}$, $L_{i^*} \succsim L_{i^*+1}$ by Axiom 3'. Additional application of Axiom 3' reveal that $L_i \succ L_{i+1}$ for $i > i^*$ and $L_i \succ L_{i-1}$ for $i < i^*$. Thus, $L_{i^*} \succsim L_i$ $\forall i \in \{0, ..., n\}$. The procedure is incentive compatible.

2.2.3 Comparing Statewise Monotonicity and Reduction

Statewise Monotone on Simple Mixtures (Axiom 3') and reduction (Axiom 3) are two fundamentally different preference assumptions. Axiom 3' is a dominance relation (the statements involve $\succ$) and Axiom 3 is a substitution axiom (the statements involve $\sim$). What complicates things further is that the types of lotteries involved in the BQSR (compound subjective lotteries) are different from the types involved in the ROE (simple mixtures). In fact, it is precisely this dichotomy that eliminates the need for reduction in the ROE.

If we were to extend the reduction axiom to assume that participants also reduce simple mixtures, statewise monotonicity would indeed be a weaker assumption than reduction. Since reporting truthfully maximizes the probability of the good outcome in each sub-lottery in the simple mixture paid by the ROE, this would also maximize the expected probability. Thus, the ROE is also incentive compatible for participants who reduce simple mixtures.

On the other hand, *the reverse is not true.* That is, the BQSR may not be incentive compatible for a participant who does not reduce the relatively simple compound lotteries involved in the BQSR, but who does have statewise monotonic preferences.

In addition, statewise monotonicity is required for participants to treat the incentives presented in multiple tasks independently within an experiment with random task payment (Azrieli et al., 2018). Because of this, any experiment with random task payment structure that has the BQSR as one of the tasks requires both Axioms 3 and 3'. Thus, within these experiments, the additional assumptions required for ROE to be incentive compatible are strictly weaker than those required for incentive compatibility of the BQSR.

3 Experimental Design

The goal of this experiment is to test whether reducers of compound lotteries report more accurate beliefs that non-reducers in a belief elicitation procedure that requires reduction for incentive compatibility; then, test whether this gap is reduced in a belief elicitation procedure that does not require reduction. To this end, the experiment is divided into two parts. Part one measures whether a participant chooses the compound lottery with the highest expected probability of winning. Part two elicits their beliefs using the two belief elicitation procedures. In part one of the experiment, participants complete two tasks. Each task requires a choice between two compound lotteries. One task, which we call the "1/3 lottery task," is the choice between (1) and (2).

$$\frac{1}{3}(75\%) + \frac{2}{3}(75\%) \tag{1}$$

$$\frac{1}{3}(56\%) + \frac{2}{3}(89\%) \tag{2}$$

The other task, which we call the "5/6 lottery task," is the choice between (3) and (4).

$$\frac{1}{6}(75\%) + \frac{5}{6}(75\%) \tag{3}$$

$$\frac{1}{6}(31\%) + \frac{5}{6}(98\%) \tag{4}$$

The order of the tasks is randomized, as well as the order (from top to bottom) of the two compound lotteries within each task.

The names of the compound lottery tasks reflect their correspondence to the lotteries the participant faces under the Binarized Quadratic Scoring Rule (BQSR) when choosing between reporting the targeted probabilities being elicited in part two of the experiment or reporting 1/2. That is, a participant who reports 1/2 in the BQSR has a 75% probability of winning the prize regardless of the outcome of the event. On the other hand, if the participant reports 1/3 in the BQSR (one of the targeted probabilities), they have a 1/3 chance of a 56% probability of winning and a 2/3 chance of a 89% probability of winning, corresponding to the two compound lotteries in the 1/3 lottery task. By eliciting these preferences over compound lotteries, we can say whether a participant faced with the choice of whether to accurately report the targeted belief under the BQSR might not do so because there exists at least one compound lottery that they prefer over the compound lottery that offers the highest expected probability of winning.

We call the participants who choose the $\frac{1}{2}(75\%) + \frac{1}{2}(75\%)$ lottery "non-reducers." The complement of the non-reducer subgroup is the subgroup who chooses the "correct" lottery when the alternative is one particular compound lottery (i.e. always have a 75% chance of winning). We refer to this complementary subgroup as "reducers," but it is important to note that a participant in this subgroup could be a non-reducer when the alternative is some other compound lottery. So, the non-reducer attribute is based on whether there is *at least one* compound lottery the participant would choose over the lottery with the highest expected probability of winning.

In part two of the lab experiment, participants complete four belief elicitation tasks. Each task is the elicitation of the participant's belief about the probability of a set of numbers being rolled by a fair, six-sided die. Two of the belief elicitations use the BQSR and two use the Rank-Ordered Elicitation (ROE). For each procedure, the two belief elicitations are for (1) a set of numbers that correspond to a $1/2$ probability (either $\{1,3,5\}$ or $\{2,4,6\}$), and (2) a set of numbers that correspond to either a $1/3$ ($\{5,6\}$) or a $5/6$ ($\{1,2,3,4,5\}$) probability. For ease of exposition, we refer to (1) as the centered probability and (2) as the non-centered probability. The instructions for each belief elicitation procedure immediately precede the two belief elicitations for that procedure. The order of the belief elicitation procedures, the set of numbers for the centered probability, the set of numbers for the non-centered probability, and the order in which beliefs are elicited within each procedure are block randomized.

We chose to elicit a centered and a non-centered probability for each procedure because the simple lottery offered by reporting $1/2$ is the starkest contrast to the compound lotteries for every other reported belief in the BQSR. For example, a participant who prefers simple to compound lotteries would still report the targeted centered probability accurately. So, we can use the comparison of behavior in reporting the centered probability to the non-centered probability to understand the mechanism.

3.1 Implementation

Participants in the experiment were recruited through Prolific.co. They were paid \$3 for completing the experiment and could earn another \$5 based on their choice in the randomly drawn task at the end of the experiment, for total potential earnings of \$8. After clicking on the link to start the experiment, participants received instructions about the general form of the experiment, their anonymity, and their payment for completing the experiment. Then, participants completed the two tasks in part one of the experiment and four tasks in part two of the experiment for a total of six tasks. The experiment concluded by informing participants which of the six tasks was randomly selected for payment and completing the randomization required to determine payment for that task. Screenshots of the full experiment are included in Appendix C. We recruited a total of 603 participants in July 2021.

4 Results

In this section, we first present the descriptive results of the experiment for the two types of tasks. Then, we test our three pre-specified hypotheses.[6] Finally, we discuss and present further results.

4.1 Descriptive Statistics

The results of the compound lottery tasks in Table 2 show that participants are close to evenly split between "always reducers" (30%), "never reducers" (30%), and "sometimes reducers" (40%). Sometimes reducers predominantly choose to reduce when faced with the 5/6 lottery task and do not reduce when faced with the 1/3 lottery task.

		1/3 Lottery Task	
		No	Yes
5/6 Lottery Task	No	29.85	9.62
	Yes	30.85	29.68

Table 2: Consistency of compound lottery preferences with reduction

Note: Each cell shows the percentage of participants whose choices were ("yes") or were not ("no") consistent with reduction in each compound lottery task. For example, the "No/No" cell gives the percentage of participants who chose the constant 75% chance of winning the $5 in both the 1/3 and 5/6 lottery tasks. All participants fall into one of the four cells, so that the percentages in the cells sum to 100%.

The results of the belief elicitation tasks in Figure 1 show that both procedures elicited beliefs that are broadly consistent with the targeted probabilities. The modal reported belief in each panel is the targeted probability. For the non-centered probabilities, the mean of each distribution is between the targeted probability and the midpoint of the support (50%). This result could reflect a pull-to-center effect,[7] or it could simply be due to the fact that a larger proportion of the support is located on the section of the support that includes 50% compared to the section of the support that does not include 50%.

[6]The AEA RCT Registry number for this study is AEARCTR-0007939.
[7]Pull-to-center has also been observed in Danz et al. (2022).

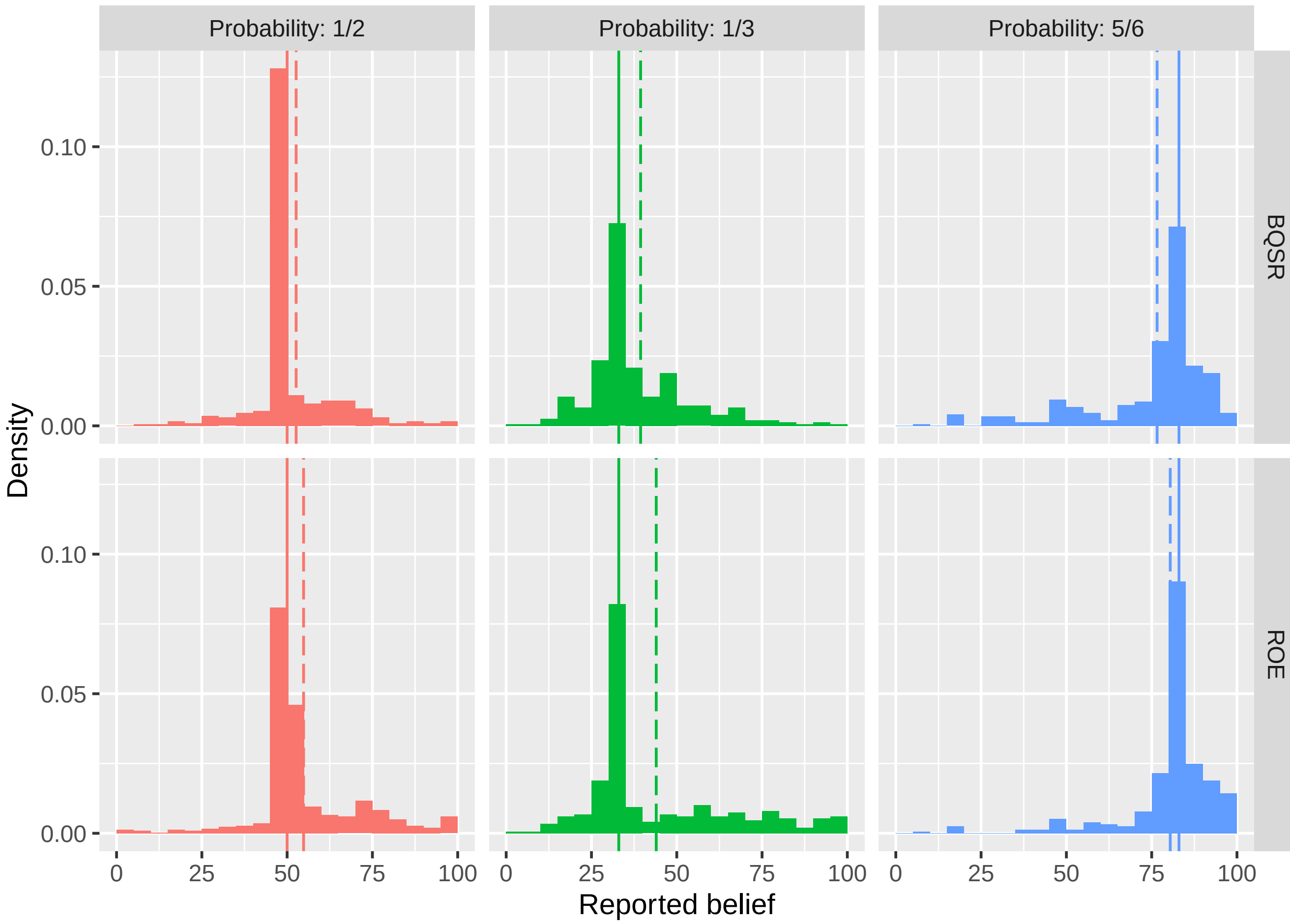


Figure 1: Distribution of reported beliefs, by elicitation procedure and targeted probability

Note: Margin labels indicate the elicitation procedure and targeted probability. Solid lines represent the targeted probability. Dashed lines represent the mean of the reported beliefs, where the ROE beliefs are assigned to the midpoint of the reported bin.

4.2 Pre-registered Hypotheses

We test three pre-specified hypotheses. The motivation for these hypotheses is that we want to learn whether the BQSR's sub-optimal empirical performance is due to a failure of the assumption that people's preferences are consistent with reduction. We first want to know whether a procedure that does not require reduction yields more accurate reporting than the BQSR. Then, whether reducers are more likely to report accurately than non-reducers in the BQSR. Lastly, we want to learn if any difference in performance between the two belief elicitation procedures is due to preferences for reduction. We use the elicitation of non-centered probabilities exclusively because, as we demonstrate in Appendix B, accurately reporting 50% in the centered elicitation is consistent with a wide variety of preferences that are inconsistent with reduction.[8]

Hypothesis 1 *The proportion of participants who accurately report the targeted non-centered probability is higher using the ROE compared to the BQSR.*

Hypothesis 2 *The proportion of reducers is higher than the proportion of non-reducers who accurately report the targeted non-centered probability using the BQSR.*

Hypothesis 3 *The difference between the ROE and BQSR in the proportion of participants who accurately report the targeted probability is larger for non-reducers than for reducers.*

Accuracy is assessed based on the relevant interval. So, when reporting the belief that a 5 or 6 is rolled by a fair, six-sided die, a participant would be counted as reporting 33% accurately if they report the interval 30-35% in the ROE and report any percentage from 30-35 in the BQSR. These intervals are dictated by the 5 percentage point bins used in the ROE.

We test Hypothesis 1 by estimating:

$$accurate_{it} = \alpha + \beta_1 [ROE]_{it} + \varepsilon_{it} \tag{5}$$

using data from the non-centered probabilities only, where $accurate_{it}$ is a binary variable equal to one if participant i accurately reports in task t and $[ROE]_{it}$ is a binary variable equal to one if the belief is elicited using the ROE. Standard errors are clustered at the participant level.

For the next two hypotheses, we define participants as either reducers or non-reducers.[9] Since the reduction of compound lotteries is only theoretically relevant to the BQSR, participants are defined based on their choice in the compound lottery task in part one of the experiment that corresponds to the non-centered belief elicited using the BQSR. That is, participants are classified as non-reducers if they were randomly assigned to report the targeted probability 1/3 using the BQSR and chose the constant 75% probability in the 1/3 lottery task. If they were instead randomly assigned to report the targeted probability 5/6 using the BQSR, participants are classified as non-reducers if they chose the constant 75% probability in the 5/6 lottery task.

We test Hypothesis 2 that reducers report more accurate beliefs using the BQSR than non-reducers by estimating:

[8]Under reduction, indifference curves are linear over the pairs of sublotteries resulting from the BQSR. For the elicitation of a 50% belief, incentive compatibility extends to preferences that are more conservative over these pairs, including any preference relation that generates convex indifference curves over these sublotteries.

[9]This definition is distinct from the always/sometimes/never reducers described at the beginning of this section.

$$accurate_i = \alpha + \beta_1[non\text{-}reducer]_i + \varepsilon_i \tag{6}$$

using data from the non-centered probability elicited using the BQSR only.

We test Hypothesis 3 that the difference in accuracy between the ROE and BQSR is due to non-reducers reporting less accurately in the BQSR compared to the ROE by estimating:

$$accurate_{it} = \alpha + \beta_1 1[non\text{-}reducer]_i + \beta_2 1[ROE]_{it} + \beta_3 1[non\text{-}reducer]_i * 1[ROE]_{it} + \varepsilon_{it} \tag{7}$$

using data from the non-centered probabilities only.

We find no evidence in favor of Hypothesis 1 that the ROE elicits more accurate reports than the BQSR. Regression results in column (1) of Table 3 show that the coefficient is the opposite of the anticipated positive difference between the ROE and the BQSR. The 95% confidence interval bounds the positive difference at 1.14 percentage points.[10] This and all other results are robust to inclusion of block-randomization stratum fixed effects, as shown in Appendix Table A.1.

On the other hand, we find strong evidence in favor of Hypothesis 2 that reducers report more accurately than non-reducers in the BQSR. Regression results in column (2) of Table 3 show that the proportion of reducers who accurately report the targeted probability is approximately 13 percentage points ($SE = 0.040$) larger than the proportion of non-reducers. This difference is a 32.5% increase over the baseline 40% of non-reducers who accurately report the targeted belief.

Our last pre-specified hypothesis was predicated on the positive difference hypothesized in Hypothesis 1; however, we found no such difference. Testing this hypothesis still allows us to learn whether there is a consistent difference between reducers and non-reducers in a procedure that does not require reduction. Regression results in column (3) of Table 3 show no difference in the differences between reducers and non-reducers when comparing the accuracy of these two subgroups in the ROE versus the BQSR. Recall that the reducer attribute is determined based on the compound lottery that corresponds to the targeted non-centered probability for the BQSR. So, the difference in differences is comparing the relative performance of two groups across the belief elicitation procedures.

4.3 Discussion & Further Results

From our pre-specified hypotheses, we learn that non-reducers report less accurate beliefs; however, using a procedure that theoretically eliminates the need for reduction has no effect on this difference in accuracy between reducers and non-reducers. The natural subsequent question is: does non-reduction directly affect the performance of

[10] Ex ante power calculations for this sample size gave us an 80% probability of detecting an effect size of 6.6 percentage points.

	(1)	(2)	(3)
Constant	0.466***	0.532***	0.532***
	(0.020)	(0.029)	(0.029)
ROE	-0.035		-0.047
	(0.024)		(0.033)
Non-reducer		-0.130***	-0.130***
		(0.040)	(0.040)
Non-reducer × ROE			0.024
			(0.047)
Observations	1,206	603	1,206
Adjusted R^2	0.000	0.015	0.013

Table 3: Accuracy for reducers and non-reducers in the BQSR and ROE tasks

Note: Estimated coefficients are from linear regressions of the accuracy dummy variable on the indicated regressors. Unit of observation is the participation-elicitation pair. Sample in columns 1 and 3 is all non-centered elicitations. Sample in column 2 is non-centered BQSR elicitations only. Standard errors clustered at the participant level in columns 1 and 3 in parentheses. Heteroskedasticity-robust standard errors in column 2 in parentheses. ***: 0.01, **: 0.05, *: 0.1.

the BQSR, or is non-reduction proxying for another characteristic like mental acumen or inattention?

To answer this question, we compare the accuracy of reducers and non-reducers in the 50% elicitation. Recall that we choose to use the elicitation of non-centered probabilities exclusively for our pre-specified hypotheses because, as we demonstrate in Appendix B, accurately reporting 50% in the centered elicitation is consistent with a wide variety of preferences that are inconsistent with reduction.

If non-reduction predicts performance in the BQSR, but the relationship is due to a non-preference characteristic correlated with preferences for reduction, then we would expect that characteristic would similarly affect people in the elicitation of the centered probability as the non-centered probability. To test this hypothesis, we estimate the following equation using the BQSR elicitations:

$$\begin{aligned} accurate_{it} = \alpha + \beta_1 1[non\text{-}reducer]_i + \beta_2 1[non\text{-}centered]_{it} + \\ \beta_3 1[non\text{-}reducer]_i * 1[non\text{-}centered]_{it} + \varepsilon_{it} \end{aligned} \tag{8}$$

If non-reducers report similarly for centered and non-centered probabilities, we would expect $\beta_3 = 0$. Instead, column 3 of Table 4 shows that we can reject that hypothesis. The gap in accuracy between reducers and non-reducers is wider in elicitations of non-centered probabilities by 8.5 p.p. ($SE = 0.043$). This result provides evidence that non-reduction plays a direct role in the performance of the BQSR.

5 Conclusion

In this paper, we conduct three pre-specified tests to study how participants' preferences with respect to the reduction of compound lotteries impact state-of-the-art belief elicitation methodology. First, we show that most participants choose compound lotteries that yield a lower expected payoff compared to their alternatives, providing a preference-based explanation for why participants often report inaccurate beliefs. Second, we show that participants who prefer the compound lotteries inconsistent with reduction are also less likely to report accurate beliefs compared to participants who choose the lotteries with the highest expected payoffs. Our third test finds, perhaps surprisingly, that a procedure that eliminates the reduction assumption does not increase accuracy. Lastly, our exploratory analysis confirms the role of reduction in the accuracy gap between reducers and non-reducers in non-centered probabilities, suggesting the importance of behavioral incentive compatibility of belief elicitation procedures.

	(1)	(2)	(3)
Constant	0.634***	0.713***	0.736***
	(0.023)	(0.018)	(0.026)
Non-reducer	-0.088***		-0.045
	(0.032)		(0.037)
Non-centered elicitation		-0.247***	-0.204***
		(0.022)	(0.030)
Non-reducer × Non-centered elicitation			-0.085**
			(0.043)
Observations	1,206	1,206	1,206
Adjusted R^2	0.007	0.062	0.071

Table 4: Accuracy for reducers and non-reducers in centered versus non-centered probabilities in the BQSR tasks

Note: Estimated coefficients are from linear regressions of the accuracy dummy variable on the indicated regressors. Unit of observation is the participation-elicitation pair. Sample is all BQSR elicitations, both centered (50%) and non-centered (not 50%). Standard errors clustered at the participant level in parentheses. ***: 0.01, **: 0.05, *: 0.1.

References

Abdellaoui, M., Klibanoff, P., and Placido, L. (2015). Experiments on compound risk in relation to simple risk and to ambiguity. *Management Science*, 61(6):1306–1322.

Azrieli, Y., Chambers, C. P., and Healy, P. J. (2018). Incentives in experiments: A theoretical analysis. *Journal of Political Economy*, 126(4):1472–1503.

Babcock, L., Recalde, M. P., Vesterlund, L., and Weingart, L. (2017). Gender differences in accepting and receiving requests for tasks with low promotability. *American Economic Review*, 107(3):714–47.

Bar-Hillel, M. (1973). On the subjective probability of compound events. *Organizational behavior and human performance*, 9(3):396–406.

Bernasconi, M. and Bernhofer, J. (2020). Catch me if you can: Testing the reduction of compound lotteries axiom in a tax compliance experiment. *Journal of Behavioral and Experimental Economics*, 84:101479.

Danz, D., Vesterlund, L., and Wilson, A. J. (2022). Belief elicitation and behavioral incentive compatibility. *American Economic Review*.

Dianat, A., Echenique, F., and Yariv, L. (2018). Statistical discrimination and affirmative action in the lab.

Hajimoladarvish, N. (2018). How do people reduce compound lotteries? *Journal of Behavioral and Experimental Economics*, 75:126–133.

Harrison, G. W., Martínez-Correa, J., and Swarthout, J. T. (2015). Reduction of compound lotteries with objective probabilities: Theory and evidence. *Journal of Economic Behavior & Organization*, 119:32–55.

Hossain, T. and Okui, R. (2013). The binarized scoring rule. *Review of Economic Studies*, 80(3):984–1001.

Machina, M. J. (1995). Non-expected utility and the robustness of the classical insurance paradigm. *The Geneva Papers on Risk and Insurance Theory*, 20(1):9–50.

Roth, A. E. and Malouf, M. W. (1979). Game-theoretic models and the role of information in bargaining. *Psychological review*, 86(6):574.

Smith, C. A. (1961). Consistency in statistical inference and decision. *Journal of the Royal Statistical Society: Series B (Methodological)*, 23(1):1–25.

Appendix A: Additional table

	(1)	(2)	(3)
ROE	-0.035		-0.047
	(0.024)		(0.033)
Non-reducer		-0.139***	-0.140***
		(0.042)	(0.041)
Non-reducer × ROE			0.024
			(0.048)
Stratum FE	Yes	Yes	Yes
Observations	1,206	603	1,206
Adjusted R^2	0.001	0.018	0.017

Table A.1: Accuracy for reducers and non-reducers in the BQSR and ROE tasks, stratum FE specifications

Note: Estimated coefficients are from linear regressions of the accuracy dummy variable on the indicated regressors and stratum fixed effects. Stratum FE correspond to the block-randomization of tasks and task ordering faced by participants. Unit of observation is the participation-elicitation pair. Sample in columns 1 and 3 is all non-centered elicitations. Sample in column 2 is non-centered BQSR elicitations only. Standard errors clustered at the participant level in columns 1 and 3 in parentheses. Heteroskedasticity-robust standard errors in column 2 in parentheses. ***: 0.01, **: 0.05, *: 0.1.

Appendix B: Additional Theory– 50% Belief

In this section, we briefly outline the theory behind our claim that many reasonable preference models are consistent with truth-telling in the $p = 0.5$ lottery elicitation under the BQSR.

In the BQSR, regardless of belief, the participant receives a compound lottery over outcomes a and b that has two simple sublotteries. These are the lotteries that occur respectively if the event is true and if it is false. Here, as before, $\tilde{p}$ is the stated belief.

$$L_T = \left(a \circ 1 - (1 - \tilde{p})^2, b \circ (1 - \tilde{p})^2\right)$$

$$L_F = \left(a \circ 1 - \tilde{p}^2, b \circ 1 - (1 - \tilde{p})^2\right)$$

Since these lotteries are binary over the same outcomes, they can be characterized by the probability of event a. Thus, for any $\tilde{p}$, L_T and L_F and thus the compound lottery above can be written as the ordered pair $\left(1 - (1 - \tilde{p})^2, 1 - \tilde{p}^2\right)$. By mapping the possible compound lotteries in the BQSR to these ordered pairs, we can apply an elementary two-good choice analogy to the participant's problem of choosing a probability in the BQSR.

Letting p_t be the probability of outcome a if the event is true and p_f be the probability of outcome a if the event is false, the budget is given explicitly by:

$$p_f = p_t + 2\sqrt{1 - p_t} - 1$$

This is a strictly concave budget symmetric about the 45-degree line and including endpoints $(1, 0), (0, 1)$ as well as the point $(0.75, 0.75)$. This is the budget faced by participants in the BQSR regardless of belief. Preferences over these bundles (p_t, p_f) depend on the participants actual belief as well as the details of their preferences about objective and subjective risk.

For instance, a participant with belief p that the event will occur believes that lottery L_t occurs with probability p and L_f occurs with probability $1-p$. A participant who treats lotteries involving subjective contingencies just like objective lotteries where the subjective events are replaced with their relevant subjective probability and also has preferences consistent with the reduction of compound lotteries is willing to trade off between p_t and p_f at a rate of $-\frac{p}{1-p}$: the odds-ratio associated with their belief. Thus, in the case of a belief $p = 0.5$, the indifference curves for (p_t, p_f) are lines with a slope of -1 since an increase to the probability of event a in L_t can be offset by the same reduction in L_f. Under the additional assumption of monotonicity, the optimal choice from the budget set is the bundle $(0.75, 0.75)$. This is, of course, consistent with "truth-telling" in the BQSR (revealing a belief of 0.5).

This demonstrates the delicate nature of the BQSR. The quadratic payoff function ensures that the trade-off between p_t and p_f is equal to the odds-ratio, which is also the rate participants who reduce compound lotteries are willing to trade off between these probabilities.

However, when reduction fails, the indifference curves are no longer linear, breaking incentive compatibility. But this is true only globally– that is for all beliefs $p \in [0,1]$. Locally, the BQSR might be incentive compatible even when reduction fails. This is particularly true at $p = 0.5$. Suppose we relax reduction and instead assume that indifference curves over (p_t, p_f) are *convex* and *symmetric*. The optimal choice remains $(0.75, 0.75)$ associated with revealing belief $\tilde{p} = 0.5$. We will prove this formally below, but the geometry of the situation is rather intuitive. A convex indifference curve that is symmetric about the 45-degree line is tangent to the concave and symmetric budget only at 0.5.

Convexity ensures that the participant prefers less extreme compound lotteries with respect to the probability of the outcome a in the two sublotteries. Symmetry of the indifference curves requires that $(\pi_1, \pi_2) \sim (\pi_2, \pi_1)$. This is a reasonable assumption *only* in the case of subjective belief $p = 0.5$. This is because the compound lotteries associated with stated beliefs $\tilde{p}$ and $1-\tilde{p}$ are the same, only changing whether the sublottery with a higher chance of a occurs when the event is true or when it is false. When the participant believes these events have the same probability, it is reasonable to assume the participant has no preference over *when* the more favorable sublottery occurs– since it occurs with the same probability.

We now prove under the assumptions that preferences over (p_t, p_f) are *symmetric*, *monotone*, and *convex*, the optimal choice is $\tilde{p} = 0.5$ or the bundle $(0.75, 0.75)$.

Proof: Consider any other choice of $p_t \in [0,1]/0.75$. Assume for contradiction that it is optimal. Since the budget set is $p_f = p_t + 2\sqrt{1-p_t} - 1$, this bundle is $\left(p_t, p_t + 2\sqrt{1-p_t} - 1\right)$. Since the budget is symmetric about the 45$-$degree line, the bundle $\left(p_t + 2\sqrt{1-p_t} - 1, p_t\right)$ is also on in the budget set. By symmetry of preferences, $\left(p_t, p_t + 2\sqrt{1-p_t} - 1\right) \sim \left(p_t + 2\sqrt{1-p_t} - 1, p_t\right)$. By convexity of preferences, the convex combination of these two bundles $\left(p_t + \sqrt{1-p_t} - \frac{1}{2}, p_t + \sqrt{1-p_t} - \frac{1}{2}\right)$ is preferred to either endpoint. Since $p_t + \sqrt{1-p_t} - \frac{1}{2} < 0.75$ for any $p_t \in [0,1]/0.75$, by monotonicity, $(0.75, 0.75) \succ \left(p_t + \sqrt{1-p_t} - \frac{1}{2}, p_t + \sqrt{1-p_t} - \frac{1}{2}\right)$. By transitivity, $(0.75, 0.75) \succ \left(p_t, p_t + 2\sqrt{1-p_t} - 1\right)$ contradicting that the chosen bundle is optimal.

Appendix C: Experiment Screenshots

Welcome

Please enter your Prolific ID:

Next

Introduction

Thank you for participating!

Please click on the audio file to listen to the instructions. You can follow along in the text below.

Note that the "Next" button on pages with instructions (like this one) will only become available after the audio file has played through. If you prefer to read the instructions, please press the button to mute the audio while it plays or silence your device.

Press Play

0:17 / 1:44

You will complete 6 tasks in this survey. The "Progress Bar" at the bottom of each page tracks how far along you are.

Confidentiality

Your participation in this survey is completely voluntary and your responses are confidential. Your name will not be published and only the research team will use the information collected. There is no risk associated with your participation in this survey. You are free to withdraw from the survey at any time with no penalty beyond the loss of potential earnings.

This survey has been reviewed by the Institutional Review Board (IRB) at Vanderbilt University to ensure your privacy is protected. You may direct any questions or concerns about this survey to the Vanderbilt IRB at (866) 224-8273. For more information, click here.

Payment

For simply completing this 15 to 20 minute survey, you will earn $3. You may earn an additional $5 based on the decisions you make and random chance.

At the end of the survey, one of the 6 tasks you complete will be randomly selected by the computer to determine whether you earn the $5 in addition to the $3 you earn for completing the survey. Each task has an equal chance of being selected. Your decisions in one task will not affect the potential payment in another task, nor the probability that a task is chosen. You will not know which task is chosen until the end of the survey, so treat each task as if it is the one that determines your payment.

Next

8%

Instructions

Press Play

0:07 / 1:29

You will now complete two tasks in which you choose between two payment options. If one of these tasks is selected for payment, the computer will then randomly select two numbers to determine your payment. *Random Number A* determines your chance of earning the $5. It will be a whole number from 1 to 6 (each number is equally likely to be chosen). *Random Number B* will be a whole number from 1 to 100 (each number is equally likely to be chosen). If *Random Number B* is equal to or less than your chance of earning the $5, which was determined by *Random Number A*, you earn the $5.

Here is an example of two payment options:

Option 1

If *Random Number A* is:	1	2	3	4	5	6
Your chance of earning the $5 is:	50%	50%	50%	50%	50%	50%

Option 2

If *Random Number A* is:	1	2	3	4	5	6
Your chance of earning the $5 is:	40%	40%	60%	60%	60%	60%

If you chose Option 1, you would have a 50% chance of earning the $5 regardless of which *Random Number A* (from 1 to 6) is selected. So, you would earn the $5 if the *Random Number B* (from 1 to 100) was 50 or less. Since the computer is equally likely to pick any number from 1 to 100, this procedure provides exactly the chance of earning the $5 described by the payment option (in this case, 50%).

If you chose Option 2, you would have a 40% chance of earning the $5 if *Random Number A* was a 1 or a 2 and a 60% chance of earning the $5 if *Random Number A* was a 3, 4, 5, or 6. If you had a 40% chance of earning the $5, you would earn the $5 if *Random Number B* was 40 or less. If you had a 60% chance of earning the $5, you would earn the $5 if *Random Number B* was 60 or less.

Next

17%

Task 1

Remember, if one of these tasks is selected for payment, the computer will then randomly select two numbers to determine your payment. *Random Number A* will be a whole number from 1 to 6 (each number is equally likely to be chosen) that determines your chance of earning the $5. *Random Number B* will be a whole number from 1 to 100 (each number is equally likely to be chosen). If *Random Number B* is equal to or less than your chance of earning the $5, determined by *Random Number A*, you earn the $5.

Select the lottery you prefer to determine your payment.

Option 1

If *Random Number A* is:	1	2	3	4	5	6
Your chance of earning the $5 is:	75%	75%	75%	75%	75%	75%

Option 2

If *Random Number A* is:	1	2	3	4	5	6
Your chance of earning the $5 is:	55%	55%	89%	89%	89%	89%

Next

25%

Task 2

Remember, if one of these tasks is selected for payment, the computer will then randomly select two numbers to determine your payment. *Random Number A* will be a whole number from 1 to 6 (each number is equally likely to be chosen) that determines your chance of earning the $5. *Random Number B* will be a whole number from 1 to 100 (each number is equally likely to be chosen). If *Random Number B* is equal to or less than your chance of earning the $5, determined by *Random Number A*, you earn the $5.

Select the lottery you prefer to determine your payment.

○ Option 1

If *Random Number A* is:	1	2	3	4	5	6
Your chance of earning the $5 is:	75%	75%	75%	75%	75%	75%

○ Option 2

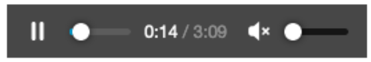
If *Random Number A* is:	1	2	3	4	5	6
Your chance of earning the $5 is:	31%	97%	97%	97%	97%	97%

Next

33%

Instructions

Press Play



In the next two tasks, you will be asked to rank one payment option relative to other payment options in order from your most preferred payment option to your least preferred payment option. The other payment options will be pre-ordered from highest likelihood of winning $5 to lowest likelihood of winning $5. We'll call this list of pre-ordered payment options the "Pre-ordered List." The one payment option you must choose how to rank is called the "Unordered Option."

If this task is selected for payment, one of the options from the Pre-ordered List will be randomly selected by the computer. Whichever you rank higher, the randomly selected option from the Pre-ordered List or the Unordered option, will determine your payment.

For example, consider an urn filled with four balls. Three of those balls are red and one of those balls is green. You could be asked to rank the following Unordered Option:

Unordered Option

- You are paid $5 if a randomly selected ball is red.

...in the following Pre-ordered List:

Pre-ordered List

- You are paid $5 with 80% chance.
- You are paid $5 with 50% chance.
- You are paid $5 with 10% chance.

You need to chose where to rank the Unordered Option relative to the payment options in the Pre-ordered List. For example, you might choose the following ranking:

1. You are paid $5 with 80% chance.
2. You are paid $5 if a randomly selected ball is red.
3. You are paid $5 with 50% chance.
4. You are paid $5 with 10% chance.

If one of the options from the Pre-ordered List determines your payment, based on your rankings and the computer's random selection, the computer will randomly select a whole number from 1 to 100 (each number is equally likely to be chosen). If that number is equal to or less than your chance of being paid the $5, you earn the $5. Otherwise, you do not earn the $5. Since the computer is equally likely to pick any number from 1 to 100, this procedure provides exactly the chance of being paid $5 described by the option.

If the Unordered Option determines your payment, based on your rankings and the computer's random selection, the computer will execute the action described in a random and fair manner. You earn the $5 if the event occurs as described in the option. In the above example, if you were to be paid based on the option "You are paid $5 if a randomly selected ball is red," the computer would randomly select a ball from the urn filled with three red balls and one green ball (each ball is equally likely to be chosen). If the ball was red, you would earn the $5. Otherwise, you would not earn the $5.

Suppose you ranked "You are paid $5 if a randomly selected ball is red" **below** "You are paid $5 with 50% chance," instead of how these two payment options are ranked in the example. Then you would have a lower chance of earning the $5 if "You are paid $5 with 50% chance." is the randomly selected option from the Pre-Ordered List. In this case, you would be paid based on a 50% chance because you ranked it higher, whereas there is a 75% chance of earning the $5 if you are paid based on a selected ball being red (3 balls out of 4). So, **you are most likely to earn the $5 if you rank the payment options from highest likelihood to lowest likelihood of occurring.**

Next

42%

Task 3

Please rank the Unordered Option on the left in the Pre-ordered List on the right by dragging the Unordered Option to a green box.

Unordered Option
You are paid $5 if a fair, six-sided die rolls a 2, 4, or 6.

Pre-ordered List
Highest-ranked
You are paid $5 with 100% chance
You are paid $5 with 95% chance
You are paid $5 with 90% chance
You are paid $5 with 85% chance
You are paid $5 with 80% chance
You are paid $5 with 75% chance
You are paid $5 with 70% chance
You are paid $5 with 65% chance
You are paid $5 with 60% chance
You are paid $5 with 55% chance
You are paid $5 with 50% chance
You are paid $5 with 45% chance
You are paid $5 with 40% chance
You are paid $5 with 35% chance
You are paid $5 with 30% chance
You are paid $5 with 25% chance
You are paid $5 with 20% chance
You are paid $5 with 15% chance
You are paid $5 with 10% chance
You are paid $5 with 5% chance
You are paid $5 with 0% chance
Lowest-ranked

Task 4

Please rank the Unordered Option on the left in the Pre-ordered List on the right by dragging the Unordered Option to a green box.

Unordered Option
You are paid $5 if a fair, six-sided die rolls a 5 or 6.

Pre-ordered List
Highest-ranked
You are paid $5 with 100% chance
You are paid $5 with 95% chance
You are paid $5 with 90% chance
You are paid $5 with 85% chance
You are paid $5 with 80% chance
You are paid $5 with 75% chance
You are paid $5 with 70% chance
You are paid $5 with 65% chance
You are paid $5 with 60% chance
You are paid $5 with 55% chance
You are paid $5 with 50% chance
You are paid $5 with 45% chance
You are paid $5 with 40% chance
You are paid $5 with 35% chance
You are paid $5 with 30% chance
You are paid $5 with 25% chance
You are paid $5 with 20% chance
You are paid $5 with 15% chance
You are paid $5 with 10% chance
You are paid $5 with 5% chance
You are paid $5 with 0% chance
Lowest-ranked

Instructions

Press Play

0:49 / 2:26

In the next two tasks, you will be asked to guess the chance that some event occurs. Your guess is a percentage probability from 0 to 100, with 0 indicating a 0-out-of-100 chance that the event occurs and 100 indicating a 100-out-of-100 chance that the event occurs. The number you provide is called *Your Guess*.

You choose *Your Guess* by clicking the response bar on your screen. The width of the green part of the bar indicates your guess that the event occurs.

- Larger values of *Your Guess* represent a greater chance that the event occurs and a smaller chance that the event does not occur.
- Smaller values of *Your Guess* represent a smaller chance that the event occurs and a greater chance that the event does not occur.

The gray part of the bar is 100 - *Your Guess*, and represents your guess that the event does not occur.

Payment Rule

We now explain how *Your Guess* is used to determine whether you earn the $5. **The payment rule is designed so that you can secure the largest chance of earning the $5 by reporting your most-accurate guess.**

- The computer chooses two numbers between 1 and 100, where each number is equally likely, as if rolling two 100-sided dice. These numbers are called Computer Number A and Computer Number B.
- The computer determines whether you earn the $5 according to whether the event occurs.

The event occurs: You will earn the $5 if *Your Guess* is **greater than or equal** to **either** Computer Number A or Computer Number B or both numbers.

The event does not occur: You will earn the $5 if *Your Guess* is **less than or equal** to **either** Computer Number A or Computer Number B or both numbers.

To help you understand the payment rule, as you move the slider to choose *Your Guess*, the computer will inform you of:

- The probability of earning the $5 if the event occurs.
- The probability of earning the $5 if the event does not occur.

For example, consider an urn filled with four balls. Three of those balls are red and one of those balls is green. You could be asked to guess whether a randomly selected ball from the urn is red. Suppose *Your Guess* was 75% and the ball selected was red. If either of the computer numbers is 75 or *lower,* then you would earn the $5. If both are *above* 75, you would not earn the $5. On the other hand, suppose your guess was 75% and the ball selected *was not* red. If either of the computer numbers is 75 *or higher,* then you would earn the $5. If both are *below* 75, you would not earn the $5.

Next

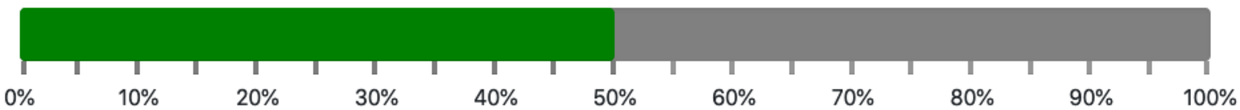
67%

Task 5

If this task is selected for payment, the computer will roll a fair six-sided die. Each number is equally likely to be rolled.

What is *Your Guess* about the chance that the die rolls a 1, 3, or 5?

Current Guess: 50%



Your probability of earning the $5 if the die roll is a 1, 3, or 5: 75%

Your probability of earning the $5 if the die roll is a 2, 4, or 6: 75%

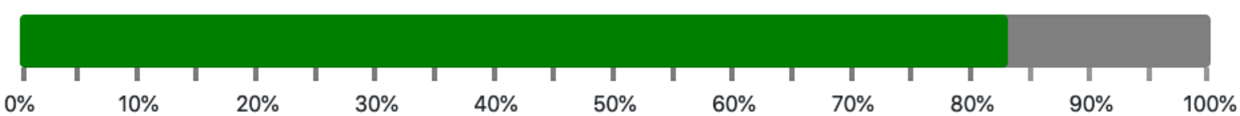
Next

Task 6

If this task is selected for payment, the computer will roll a fair six-sided die. Each number is equally likely to be rolled.

What is *Your Guess* about the chance that the die rolls a 1, 2, 3, 4, or 5?

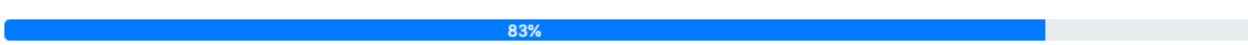
Current Guess: 83%



Your probability of earning the $5 if the die roll is a 1, 2, 3, 4, or 5: 97%

Your probability of earning the $5 if the die roll is a 6: 31%

Next



Payment

Thank you for participating! You have earned $3 for completing the survey. Now, click the button below that says "Choose Task" to determine which of the 6 tasks is randomly selected by the computer to determine your payment today.

Choose Task

92%

Payment

Task 1 has been selected for payment.

In Task 1, you indicated you prefer the following payment option:

If *Random Number A* is:	1	2	3	4	5	6
Your chance of earning the $5 is:	75%	75%	75%	75%	75%	75%

Random Number A is 4, so your chance of winning $5 is 75%. Random Number B will be a number between 1 and 100. You will win $5 if Random Number B is 75 or less. Click the button below to randomly determine Random Number B.

Random Number B is 87, so you do not win $5. You earned $3 simply for participating, so your total payment is $3.

Click "Next" for instructions on how to record your payment.

Next

92%

Please click the link below to finish the survey.

https://app.prolific.co/submissions/complete?cc=107C59DA

You will be redirected to the Prolific website and your payment will be recorded.

100%